Performance Evaluation of Phase Estimation Algorithms in Equalized Coherent Optical Systems

L. M. Pessoa, H. M. Salgado, and I. Darwazeh

***Abstract*—We assess the performance of a coherent optical system employing a phase estimation algorithm combined with a least mean square decision-directed equalizer in a feedback configuration and compare it to the constant modulus algorithm with an independent carrier phase estimation approach for 16-quadratic-amplitude-modulation transmission. Both series and parallel implementations are discussed. Hardware issues regarding a recursive implementation of the estimation filter are also reported.**

***Index Terms*—Coherent systems, feedforward phase estimation, Kalman filtering, parallel algorithms, Wiener filtering.**

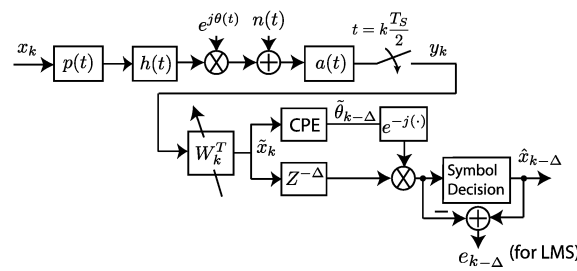

Fig. 1. System canonical model.

I. Introduction

Coherent optical communications have gained renewed interest due to the availability of high-speed digital signal processing, enabling operation on signals proportional to the electric field. Furthermore, in order to avoid the difficulties associated with the optical phase-locked loop (PLL), carrier synchronization can be done digitally, allowing for a free-running local oscillator (LO), while tolerating 50%–100% wider laser linewidth than PLL [1].

The constant modulus algorithm (CMA) is the most used blind adaptive equalization algorithm, essentially because of its robustness and ability to converge prior to phase recovery [2]. In order to cope with laser phase noise, an elegant solution consists in using the CMA for initial adaptation, thus avoiding training sequences and enabling subsequent independent carrier phase estimation (CPE). Once equalizer convergence has been achieved, there is benefit in switching to decision-directed (DD) mode, driven from symbol decision errors, which is least mean squares (LMSs) based, thus improving the demodulator SNR performance [2]. However, at this point, the phase must be estimated and its value considered in the error signal, precluding the use of independent CPE. Additionally, the equalizer decision feedback might be critical in high-speed parallelized DSP, which would lead us to use CMA. On the other hand, CMA is not optimized for a 16-quadratic-amplitude-modulation (QAM) multiple modulus constellation. In this letter, we assess the performance of a phase estimation algorithm combined with a DD equalizer in a feedback configuration and compare it to the approach consisting of CMA followed by CPE, both for series and parallel implementations. A recent work [3] suggests the integration of an adaptive digital equalizer with carrier synchronization using decision feedback. Although this concept has been suggested in [2] and [3], a detailed study on the algorithms' performance has never been done, specially taking into account the parallelization. The actual simulation of the 16-QAM coherent optical system was carried out in MATLAB.

This letter is organized as follows. In Section II, the simulation model is described; Sections III and IV present CPE issues and the proposed algorithm, and in Section V, we present and discuss the simulation results; finally, the conclusions are given in Section VI.

II. Simulation Model

A. Model Description

Fig. 1 represents the canonical model of a single polarization coherent optical system, where $p(t)$ is the pulse shape and $h(t)$ represents the fiber impulse response, which might include the effects of chromatic dispersion (CD) and polarization-mode dispersion (PMD). The signal is noise-loaded, with both phase noise $\theta(t)$ and AWGN noise $n(t)$. Phase noise is usually characterized as a Wiener process, being modeled as in [1]. The sampling occurs at a rate of 2 samples per symbol, which was previously shown to be sufficient. Actually, 1.5 samples per symbol in conjunction with a fifth-order Butterworth antialiasing filter—$a(t)$—was shown to allow for a penalty of less than 2 dB [4]. Linear equalization follows by performing a convolution with a complex-valued $T_S/2$ spaced finite-impulse response (FIR) filter. The linear equalizer calculates $\tilde{x}_k$, which is the minimum mean-squared-error estimate of the kth transmitted symbol x_k. The optimum solution for the coefficients is obtainable by the Wiener–Hopf equation [4]. Furthermore, we consider that the channel may vary with time due to PMD, so its frequency response is not exactly known and an adaptive

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equalizer is desirable [4]. Therefore, the LMS or recursive least-squares (RLS) algorithms can be used to continually adjust the coefficients. The LMS coefficient update equation is given by

$$W_{k+1} = W_k + \mu \cdot \left(e^*_{k-\Delta_1}\right) \cdot \left(y_{k-\Delta_1}\right) \tag{1}$$

where W is the equalizer coefficient vector, μ is the algorithm step size, e_k is the conjugated error, y_k is the equalizer input vector [3], and Δ_1 is an arbitrary delay that might be greater than zero if the required coefficient update rate is lower than the symbol rate due to slowly varying impairments such as PMD. The error signal is calculated differently for LMS and CMA; the actual detailed formulas are available in [5]. A practical implementation of this system could use a fixed long filter for CD compensation of long fiber distances, and then a subsequent small filter with rapid updates (13-tap of length has been used by several authors [5]) is enough to mitigate the residual uncompensated CD and dynamic polarization changes. This concept has been validated in [5] for both LMS and CMA.

III. Carrier Phase Estimation

Assuming the signal at the output of the equalizer is perfectly compensated, it is only impacted by phase noise and AWGN

$$\tilde{x}_k = W_k^T y_k = x_k e^{j\theta_k} + n_k \tag{2}$$

where θ_k is the carrier phase and n_k is AWGN. The goal of the phase estimation process is to obtain $\tilde{\theta}_k$, an estimate of θ_k, which will allow derotation of the signal by multiplying it with $e^{-j\tilde{\theta}_k}$, followed by a symbol-by-symbol detector to find $\tilde{x}_k$, the estimate of x_k. The soft estimate ψ_k is the phase of $\tilde{x}_k$ referenced to the phase of x_k

$$\psi_k = \theta_k + n'_k \tag{3}$$

where $n_{k'}$ is the projection of n_k onto a vector orthogonal to $x_k e^{j\theta_k}$. This estimate can be obtained either through DD or nondecision-aided (NDA, known as Mth power) approaches. Although NDA for 16-QAM has been proposed in [6], the midamplitude symbols (named class II) cannot contribute to the estimate because of the irregular phase spacing, where the remaining symbols, belonging to class I, are used. Additionally, long sequences of class-II symbols have to be prevented with appropriate line codes (to avoid cycle slips), and an amplitude correction of the inner symbols must be performed for optimal performance. However, this has only been done for the simple running average filter. In the following sections, the performance of the Wiener filter is also discussed.

IV. Proposed Algorithm

The Wiener filter was proposed in [1] as the optimal filter for CPE, which can be approximated by an FIR filter with N taps (typically 10–100), in such a way that coefficients that are less than 5% of the largest are neglected [1]. This filter consists of two symmetric exponentially decaying sequences, causal and anticausal, with an inherent optimum delay of $N/2$. For the case of combined DD equalization and CPE, we have found through simulation that optimum performance is obtained for zero delay, because of phase feedback, if the equalizer update rate equals the symbol rate. This stems from a compromise between equalizer delay and CPE accuracy. Here, we propose the Kalman filter, which is a recursive (IIR) implementation of the Wiener filter, to implement the zero-delay filter, with the advantage of reducing the computational complexity. For DD phase estimation, the soft estimate error is given by

$$\begin{aligned}\psi_k - \tilde{\theta}_k &= \text{angle}\,(\tilde{x}_k) - \text{angle}(d_{\text{ref}}) - \tilde{\theta}_k \\ &= \text{angle}\,(\tilde{x}_k e^{-j\tilde{\theta}_k} \cdot \text{conj}(d_{\text{ref}})).\end{aligned} \tag{4}$$

Including the result of (4), the Kalman recursion is given by

$$\begin{aligned}\tilde{\theta}_{k+1} &= \tilde{\theta}_k + G \cdot (\psi_k - \tilde{\theta}_k) \\ &= \tilde{\theta}_k + (1-\alpha) \cdot \text{angle}\,(\tilde{x}_k e^{-j\tilde{\theta}_k} \cdot \text{conj}(d_{\text{ref}}))\end{aligned} \tag{5}$$

where d_{ref} is the output of a decision device when its input is $\tilde{x}_k e^{-j\tilde{\theta}_k}$, $\alpha = (1 + r/2) - \sqrt{(1 + r/2)^2 - 1}$, and $r = \sigma_p^2/\sigma_{n'}^2$ [1] is the ratio between the magnitude of phase noise and AWGN. The parameter r determines the rate of decay of the filter coefficients, and we calculate it iteratively, as the SNR might be unknown. This might be a disadvantage for the Kalman approach, if the convergence of the estimation takes too long. Furthermore, recursive filters are sensitive to both parallelization and quantization issues. In Fig. 2(a), we show that at least 8 bits are required in order to avoid performance degradation above 2 dB stemming from fixed-point quantization, while above 10 bits, both FIR and IIR tend to the same performance. The discussed algorithms should be implemented with a high degree of parallelism, otherwise we are unable to implement them in real time with the currently available technology. The algorithms might be modified to accomplish this, with a look-ahead computation [7] to refer the feedback to a result obtained L symbols before, at the expense of extra feedforward (FF) taps. The Kalman recursion to obtain the estimated phase is given by

$$\tilde{\theta}_{k+1} = (1-\alpha) \sum_{n=0}^{L-1} \alpha^n \psi_{k-n} + \alpha^L \tilde{\theta}_{k-L+1}. \tag{6}$$

The calculation of $\tilde{\theta}_{k+2}$ can be started before the previous $\tilde{\theta}_{k+1}$ is complete, which supports the parallel implementation. In this way, cycle slips will have the same impact as in a serial system.

V. Results

Fig. 2(b) compares the BER performance versus linewidth per bit rate for different phase estimation algorithms. A non-Gray differential bit encoding scheme (proposed in [1]) was employed in all cases, preventing catastrophic bit error propagation when phase noise is high. It is clear that the running average filter using NDA is not as optimized as Wiener (specifically IIR) approaches. The DD approach with $L = 32$ is always worse than NDA, which shows that DD is not well suited for parallelization. We should emphasize that the NDA approach might be parallelizable with no loss since it involves no CPE feedback. Fig. 3 compares the performance of several scenarios involving both equalization and phase estimation, including a third-order low-pass Bessel filter for $p(t)$ and $a(t)$. The results were found through the Monte Carlo technique and evaluated after convergence with a step size of $\mu = 10^{-3}$. Although CMA is not optimized for 16-QAM, it still converges at a cost of a higher penalty than LMS (1.2 dB opposed to 0.1 dB, respectively, at low linewidths). In the result represented by "$*$," the phase is estimated at the output of the LMS-DD equalizer, with

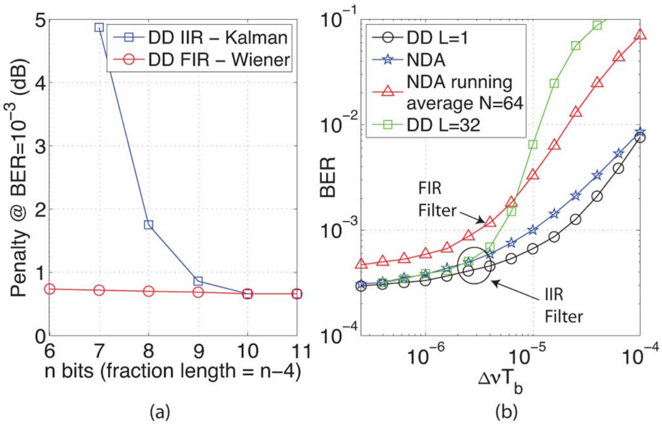

Fig. 2. (a) Penalty for a target BER $= 1e-3$. $\Delta\nu T_b = 10^{-5}$. (b) BER versus laser linewidth per bit rate. $Eb/N0$ of 12 dB (1 dB above the differential AWGN limit BER $= 1 \times 10^{-3}$).

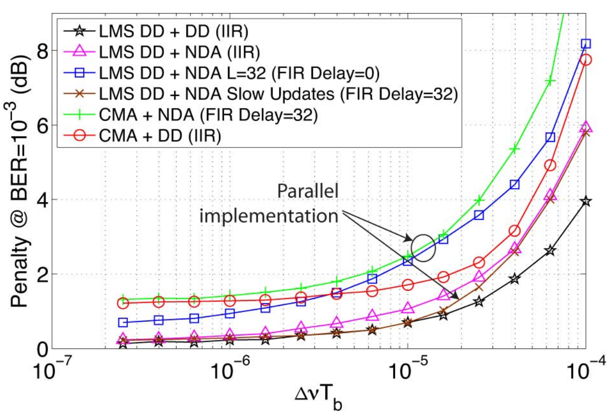

Fig. 3. Penalty versus laser linewidth per bit rate. Reference E_b/N_0 of 12 dB (BER $= 1 \times 10^{-3}$ sensitivity point for LMS).

the IIR-DD CPE filter, and then fed back in the error signal. In curves "△" and "□," the CPE was changed to IIR-NDA with $L = 1$ and FIR-NDA with $L = 32$, respectively. In the latter, IIR was still employed for the other $N - L$ taps, as suggested by (6). Curve "×" refers to the LMS-DD equalizer with slow updates ($\Delta_1 \gg 32$), using the optimized CPE filter with delay $= 32$. The observable difference to "△" is due to the CPE delay only. Additionally, it might be parallelized with no penalty due to the almost insignificant equalization feedback. The "+" and "o" results correspond to separate CMA equalization and subsequent CPE through the IIR-DD and FIR-NDA approaches, respectively. While the former is not efficiently parallelizable as we have seen before, the latter may be parallelized and can benefit from using a finite delay while not causing any penalty due to the absence of equalization feedback. It is relevant to note that we named FIR the filters used for the parallel versions because the L FF taps are intrinsic to the parallel implementation. Depending on the maximum speed of available DSP chips, the required parallelization level will dictate which approach should be chosen. If no parallelization is required, the LMS-DD should be chosen based on a good compromise between complexity and performance. If a parallelization level $L \leqslant 32$ is necessary, both approaches have similar complexities, with the LMS + NDA approach being preferable based on performance, but if the requirement is greater than 32, then CMA + NDA is the best option, because its performance does not depend on L. The major strengths and weaknesses of the several approaches are summarized in Table I.

TABLE I
COMPARISON OF DIFFERENT EQUALIZER/CPE APPROACHES

Eq.	CPE Filter	Strengths	Weaknesses	
LMS-DD	DD	Small penalty, specially for small linewidths. IIR filter can be used, reducing the computational complexity. Phase unwrapping is more robust than for NDA.	Not Parallelizable due to CPE Feedback.	Equalizer Feedback. When used, IIR is susceptible to feedback issues.
	NDA Serial / Parallel	No CPE Feedback. IIR filter can still be used in the non-feedforward part of the filter tail. If slow updates are allowed, it is the best option for parallel processing.	FF filter section needed for parallel implementation. For L>32, CMA approaches are preferable.	
CMA	DD	Smaller penalty than CMA+NDA for large linewidths. IIR filter can be used, reducing the computational complexity. Phase unwrapping is more robust than for NDA.	Not Parallelizable due to CPE Feedback. Higher penalty than LMS-DD+DD.	CMA equalizers can not achieve as low MSE as LMS based, specially for the 16-QAM non-constant modulus constellation.
	NDA Serial / Parallel	No CPE Feedback. Penalty is independent from L. IIR filter can still be used in the non-feedforward part of the filter tail.	1dB worse than LMS-DD+NDA. FF filter section needed for parallel implementation.	

VI. CONCLUSION

The performance of a phase estimation algorithm operating in feedback with an LMS-DD equalizer in coherent optical systems was assessed both for DD and NDA CPE. Its performance was compared to the CMA having separate CPE. Parallelization issues were also addressed. It was found that up to $L = 32$, the LMD-DD using equalization feedback still overperforms the equivalent CMA approach, while having similar computational complexity levels. Additionally, if the equalizer updates can be slow, LMS-DD + NDA is always the best for parallel processing.

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