Thesis Research Plan

Mitigation of Fiber Impairments in Coherent Optical Systems

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Abstract

This document details the current state of work done under thesis investigation, and addresses the directions of future investigation. Impairments mitigation has greater potential in coherent systems because of the linear conversion from the optical to the electrical domain, specially when polarization multiplexing is explored as a means of increasing spectral efficiency. The implementation of digital signal processing algorithms for mitigation of chromatic dispersion (CD), polarization mode dispersion (PMD) and laser phase noise in a coherent optical receiver exploiting polarization multiplexing are investigated. Future research will focus on optimization techniques and algorithms for reduction of the impact of nonlinear effects in coherent optical systems.
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Chapter 1

Introduction

1.1 Background and Motivation

Recently, the main challenge faced by optical engineers has been increasing the throughput and distance limit of existent long-haul transmission systems, without doing signal regeneration. Coherent systems are seen today as the key in order to accomplish these requirements. These systems were a topic of intense research during the 80’s, essentially because of their great sensitivity and capability of narrowband channel selection, until the emergence of Erbium Doped Fiber Amplifiers (EDFA) in the early 90’s. Recently, they have gained renewed interest, essentially because of the availability of high speed digital signal processing, which allows for the signal to be digitized and processed in the digital domain. The lower price of electrical components, partly relaxed receiver requirements at high data rates and capability of pushing the spectral efficiency limits beyond, while maximizing the power efficiency, also intensified the interest in the topic. In fact, if the outputs of a coherent homodyne receiver are sampled at the Nyquist rate, the digitized waveform contains full information of the electric field, preserving the amplitude, phase and polarization from the optical domain to the electrical domain, enabling new potential of multi-level signaling (M-ary PSK and M-ary QAM modulation), as well as the possibility of exploring polarization multiplexing [1]. Therefore the symbol rate can be reduced while keeping the bit rate, increasing the spectral efficiency and easing the complexity of A/D circuits used in demodulation/compensation schemes. Additionally, it enables quasi-exact compensation of linear transmission impairments such as
Chromatic Dispersion (CD) and Polarization Mode Dispersion (PMD) by a linear filter [2], which can operate adaptively to overcome time-varying impairments. Having said this, we can state the main objective of this work: investigate the compensation of fiber impairments in coherent optical systems by electronic domain equalization using digital processor technology. Specific milestones are outlined below:

- Study of phase estimation algorithms suitable for implementation in digital signal processors.
- Investigation of limited length equalizers for dispersion compensation in the electrical domain in coherent optical systems.
- Simulation and design of equalizers based on the transversal filter technique for operation at rates above 40 Gb/s.
- Through simulation and experimentation assess the efficacy of the equalizers.
- Conduct full studies of transmission impairments including fibre non-linear effects and assess the performance, both theoretically and experimentally, of coherent systems with compensation in the electronic domain.

1.2 Relevance of the subject

The impairments associated with currently installed fiber become significant at high data-rates and fiber lengths. Therefore, in order to cope with the bandwidth restrictions of optical fibers and amplifiers, maximizing the system spectral efficiency turns out to be mandatory. Moreover, the power efficiency should also be maximized in order to avoid fiber nonlinearities, which corresponds to minimizing the required transmitted energy per bit. DSP emerges in this context as a flexible tool to allow the manipulation of the signal in the digital domain, in order to compensate for all the distortion caused by the fiber channel (linear and non-linear) as well as distortions stemming from the lasers used in both transmitter and receiver ends. Therefore, the algorithms to be developed will take even further both the data-rates and system reach of currently deployed networks. These requirements are expected to become even more stringent with time due to the growing demands on broadband
services, such as high definition multimedia contents. DSP has been evolving as a practical solution for robust optical long-haul transmission, and it is expected that data converters will be able to satisfy the required high sampling rates in the near future. In 2007, a 90 nm 20 M gate ASIC with 4 integrated ADCs (> 20 GSa/s) has been reported [4]. On the other hand, as soon as sufficiently high speed data converters are available, field programmable gate arrays (FPGA) propose to be a very flexible and fast time to market design tool, to implement the developed algorithms.

1.3 Structure of the document

This document is organized as follows: Chapter 3 presents the state-of-the-art of the topic, in chapter 4 we present and discuss some preliminary developments including proposed algorithms and simulation results; finally the future research plan will be detailed in chapter 5.

1.4 Publications / Contributions


Chapter 2

State-of-the-art

2.1 Introduction

The most common type of optical communication system are Intensity Modulation / Directed Detection (IM/DD) based, essentially due to their cost, simplicity and effectiveness. However, dispersion compensation in IM/DD systems is not very efficient due to the non-linear optical to electrical (O/E) conversion in the photodiode, with loss of phase information. However, single side band (SSB) transmission allows the square law detection to largely preserve the phase. Moreover, maximum likelihood sequence estimation (MLSE) is the most effective means of impairments mitigation in these systems [3].

In contrast to IM/DD systems, complete equalization of transmission impairments is possible in coherent systems, in the electrical domain, as the equalizer operates on signals proportional to the electric field. Additionally, zero penalty dispersion compensation may also be achieved in the optical domain. However, adaptive schemes are rather complicated because the error signal is obtained after square-law detection in the photodiode [3].

Considering coherent receiver implementation, homodyne receivers are superior to their heterodyne counterparts, and seem to be the choice for future networks. However, as they are sensitive to phase noise, an elegant technique called phase diversity emerged, but only applicable to 2-level modulation signals. Therefore, for higher order modulation signals, an optical phase locked loop (OPLL) is necessary, which nowadays is still very difficult to implement. A turnaround to this problem is the usage of DPSK modulation, where the information is encoded by changes in phase from one symbol
to the next, and differential detection, which consists in pair-wise comparison of sample phases, assuming the optical carrier phase varies much more slowly than the phase modulation. However, this detection scheme is less performing than synchronous detection [5], where the decoding of data is performed on the basis of comparison of consecutive quadrant numbers, but requiring that the phase of the signal is tracked. Thereafter, the option is to cope with phase noise through digital phase estimation, using a DSP to track the signal phase [6].

In recent experiments, the typical receiver is based on a phase and polarization diversity configuration [7]. Considering that the Local Oscillator (LO) phase needs to be locked to the signal phase, to avoid the difficulties associated with the OPLL, the synchronization can be done in the DSP, by digital phase estimation techniques, allowing for a free running LO.

The main goal of the algorithms used in coherent receivers is to perform equalization and phase recovery. Equalizers can achieve adaptation of their coefficients either by transmitting a training sequence, known symbol statistics or decision-directed (DD) adaptation. When no training sequence is transmitted, the operation is referred to as blind equalization and the constant modulus algorithm (CMA) introduced by Godard and Treichler is the most used, essentially because of its robustness and ability to converge prior to phase recovery [8]. However, for non-constant modulus constellations as QAM, the multimodulus algorithm (MMA) introduced by Yang et al improves the performance of CMA by obtaining low steady-state mean-squared error (MSE) [9], but its cost function is not carrier phase independent. In order to cope with laser phase noise, an elegant solution consists in using the CMA for initial adaptation, avoiding bandwidth consuming training sequences, and enabling subsequent independent carrier phase estimation (CPE). Once equalizer convergence has been achieved, there is a great benefit if the equalizer switches to DD mode, whereby the error signal is derived from the error between the baseband signal and the nearest, ideal point of the constellation, improving the demodulator SNR performance [8]. However, at this point, the phase must be estimated and its value considered in the error signal, precluding the use of independent CPE.
2.2 Theoretical considerations

The compensation of fiber impairments in the digital domain, in coherent optical systems, means that, in principle, any linear distortion can be compensated in the DSP at 1 sample/bit, as long as an analog matched filter precedes the sampler. However, the matched filter has several drawbacks: It may be more difficult to design than its digital equivalent and the exact phase of sampling is required to be known to sample the signal at its maximum energy. Moreover, if the channel response is unknown or time-varying, an adaptive equalizer is necessary whereas an adaptive analog matched filter may be difficult to design. Even if the sample time is optimum, spectral overlap always occurs unless the system transfer function is a sinc function, which is not realizable in practice. A fractionally spaced equalizer (FSE) implements the matched filter and equalizer as a single unit [10].

Figure 2.1: Coherent transmission system employing a phase and polarization diversity homodyne receiver. LD - Laser Diode, PBS - Polarization Beam Splitter, OA - Optical Amplifier, BD - Balanced Detector.

Figure 2.1 represents a diagram of the coherent transmission system under study. In the transmitter optical IQ-Modulators are used whereby the laser signal light is split into two orthogonal carriers and then modulated by Mach-Zehnder-Modulators (MZM), biased at minimum transmission, and driven by multi-level RF signals. For 16-QAM these RF signals have 4-levels. Then, the two data streams, polarized in orthogonal directions, are combined in a Polarization Beam Splitter (PBS) and launched into the transmission channel (optical fiber). The signal is amplified to overcome attenuation in the long haul transmission. Additive White Gaussian Noise (AWGN) comes from the amplified spontaneous emission (ASE) of optical amplifiers which dominates over LO shot noise [11]. The optical multi-level modulation signal can be de-
ected by an homodyne IQ-receiver, whose general configuration is valid for any M-PSK and M-QAM modulation format. Coherent detection involves beating the incoming signal with light from a local oscillator (LO) laser [12], of near-identical wavelength and similar state of polarization (SOP), generating a photocurrent in the detector that corresponds to the beat product of the two lightwaves. Figure 2.1 shows how the signal is mixed with the LO in a phase/polarization diverse hybrid. We can see that no polarization controllers are present, due to the polarization diversity configuration, where both signal and LO waves are separated by polarization beam splitters (PBS) into orthogonal components, each going to a separate 90-hybrid, where the signal is coherently detected using four balanced photodiodes. The four electrical output signals correspond to the I and Q components associated with the parallel and orthogonal LO polarizations. Finally, the signals are low-pass filtered by anti-aliasing filters $a(t)$ and sampled at the Nyquist rate.

The following equations will give a mathematical support to the previous explanation. In these, we consider only one phase diverse hybrid and assume that the signal is aligned in SOP with the LO in that hybrid. The complex envelope of the incoming optical multi-level modulation signal, and local oscillator laser are:

$$E_S(t) = a(t) \cdot e^{j \phi(t)} \cdot \sqrt{P_S} \cdot e^{w_{st}t + \phi_S + \phi_{NS}} =$$

$$= [I(t) + jQ(t)] \cdot \sqrt{P_S} \cdot e^{w_{st}t + \phi_S + \phi_{NS}} =$$

$$= E_I(t) + jE_Q(t);$$

$$E_{LO} = \sqrt{P_{LO}} \cdot e^{w_{LO}t + \phi_{LO} + \phi_{NLO}} \quad (2.1)$$

where $w_S$ and $w_{LO}$ are the angular frequencies of the signal optical carrier and local oscillator, $P_S$ and $P_{LO}$ the CW power, $\phi_S$ and $\phi_{LO}$ initial phases, and $\phi_{NS}$ and $\phi_{NLO}$ the phase noise of the signal and LO lasers. $I(t)$ and $Q(t)$ are the in-phase and quadrature components of the transmitted complex envelope, and $a(t)$ and $\phi(t)$ its magnitude and phase. The signal wave and the LO wave combine in an optical $2 \times 4$ 90-hybrid, yielding 4 output fields. The electric field components (EFCs) at the output of the hybrids are detected by means of balanced photodiodes, which has the advantage of suppressing the relative intensity noise (RIN) [3]. The in-phase and quadrature photocurrents are
\[ I^*(t) = \gamma \cdot (I(t) \cdot \cos(\Delta \phi) + Q(t) \cdot \sin(\Delta \phi)) \] (2.3)

\[ Q^*(t) = \gamma \cdot (-I(t) \cdot \sin(\Delta \phi) + Q(t) \cdot \cos(\Delta \phi)) \] (2.4)

with \( \gamma = R \cdot \sqrt{P_{LO}P_S/2} \), when neglecting shot noise. In the above equations, \( R \) is the responsivity of the photodiodes, and \( \Delta \phi \) is the phase error due to frequency offset, phase offset and laser phase noise, which is given by

\[ \Delta \phi(t) = (w_S - w_{LO})t + (\phi_S - \phi_{LO}) + (\phi_{NS} - \phi_{NLO}) \] (2.5)

Therefore, in order to extract the modulation information the total phase error must be controlled. Moreover, for zero phase error, the in-phase and quadrature components of the transmitted complex envelope are obtained separately in the two arms. Signals \( y_{i,1} \) and \( y_{q,1} \) are the in-phase and quadrature components of the received EFC \( \hat{E}_x(t) \), like \( y_{i,2} \) and \( y_{q,2} \) are for the EFC \( \hat{E}_y(t) \). These signals are then sampled, with a sampling interval of one half the symbol period \( T_s/2 \), to allow the employment of a \( T/2 \) fractionally spaced equalizer (FSE) in the digital domain, which is optimum for coherent receivers [13].

### 2.3 Model description

![System canonical model](image)

Figure 2.2: System canonical model

Rather than dealing with the complexities associated with practical implementations of coherent transmission systems, simulation platforms are a very effective way to allow the assessment of different fiber impairments mitigation algorithms. Therefore we have investigated several issues concerning
the modeling of the whole transmission system. The figure above represents
the canonical model of a coherent optical system employing polarization mul-
tiplexing. The transmitted signals, in each polarization are given by:

\[ x_l(t) = \sum_n x_{l,n} p(t - n T_s) \]  \hspace{1cm} (2.6)

where \( x_{l,n} \) is the \( n \)th symbol transmitted in the \( l \)th polarization and \( p(t) \) is
the transmitted pulse shape. The received signal is then of the form:

\[ \begin{bmatrix} r_1(t) \\ r_2(t) \end{bmatrix} = \left( \begin{bmatrix} h_{11}(t) & h_{12}(t) \\ h_{21}(t) & h_{22}(t) \end{bmatrix} \otimes \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \right) \cdot e^{j\theta(t)} + \begin{bmatrix} n_1(t) \\ n_2(t) \end{bmatrix} \] \hspace{1cm} (2.7)

The matrix \( H(t) \) represents the fiber impulse response, completely
describing a dually polarized channel, which is mathematically represented as a
2x2 Multiple Input Multiple Output (MIMO) channel [4]. The frequency
response of the fiber can be described by:

\[ H(w) = T(w) \times e^{-\frac{1}{2} \beta_2 L w^2} \] \hspace{1cm} (2.8)

where \( \beta_2 \) is the Group Velocity Dispersion (GVD) parameter and \( L \) is the
fiber length. Chromatic dispersion is accounted up to second order. \( T(w) \) is
the fiber Jones Matrix accounting for PMD, given by:

\[ T(w) = \begin{bmatrix} u_1(w) & u_2(w) \\ -u_2^*(w) & u_1^*(w) \end{bmatrix} \] \hspace{1cm} (2.9)

where \( |u_1(w)|^2 + |u_2(w)|^2 = 1 \). The numerical modeling of the Jones matrix is
obtained by a concatenation of unequal sections of birefringent fiber, which
can be expressed as [14]:

\[ T(w) = \prod_{n=1}^{N} B_n(w) R(\alpha_n) \] \hspace{1cm} (2.10)

where

\[ B_n(w) = \begin{bmatrix} e^{j\frac{1}{2} \sqrt{\frac{1}{8} w D_{PMD} + \phi_n}} & 0 \\ 0 & e^{-j\frac{1}{2} \sqrt{\frac{1}{8} w D_{PMD} + \phi_n}} \end{bmatrix} \] \hspace{1cm} (2.11)

\[ R(\alpha_n) = \begin{bmatrix} \cos \alpha_n & \sin \alpha_n \\ -\sin \alpha_n & \cos \alpha_n \end{bmatrix} \] \hspace{1cm} (2.12)
where \(N\) is the number of fiber segments \((N = 80 \text{ in our model})\), and \(B_n(w)\) represents the birefringence matrix of the \(n^{th}\) segment of length \(h_n\). \(R(\alpha_n)\) is the matrix of a rotator that represents the random coordinate transformation of the birefringent segment axes, producing therefore, a frequency independent differential group delay (DGD) in each section. The phase angle \(\phi_n\) accounts for the small temperature fluctuation along the fiber, being a stochastic variable with a uniform distribution between 0 and \(2\pi\). \(D_{PMD}\) is the PMD coefficient of the fiber in \(\text{ps}/\sqrt{\text{km}}\). For a given total PMD \(\langle \Delta \tau \rangle\) and fiber length \(L\), the size of each segment was randomly generated from a Gaussian distribution around the mean length \(h_n = L/N\) with standard deviations \(\Delta h\) varying from 0 – 30% of the mean length, in order to produce non-periodic variations of the DGD over the frequency spectrum, and achieve a Maxwellian distribution. Furthermore, as the variation of \(T(w)\) as a function of frequency is very slow on the length scale given by \(h_n\), it is only necessary to evaluate it at a few frequencies, intermediate values being obtained through interpolation [15].

After adding the channel distortion effect, the signal is noise loaded, with both AWGN noise and phase noise. Phase noise is usually characterized as a Wiener process, described in the discrete domain as:

\[
\phi_k = \sum_{-\infty}^{k} \nu_m
\]

where the \(\nu_m\)'s are independent identically distributed (i.i.d.) Gaussian random variables with zero mean and variance \(\sigma_p^2 = 2\pi \Delta \nu T\). \(\Delta \nu\) is generally assumed to be the 3-dB combined linewidths of the signal and LO lasers (also known as the beat linewidth), and \(T\) is the symbol period [16]. In our numerical model, the transmitter laser is assumed to be ideal, whereas the LO laser is assumed to have phase noise equal to the sum of the linewidths of the two lasers [2].

Letting \(g(t) = a(t) \otimes h(t) \otimes p(t)\) and \(n'_l = a(t) \otimes n_l(t)\) we can write the signal after the anti-alias filters as:

\[
y_l(t) = \sum_{n} \sum_{m=1}^{2} x_{m,n} q_{lm}(t - nT_s) e^{j\phi(t)} + n'_l(t) \tag{2.13}
\]

The sampling occurs at a rate of 2 samples per symbol, which was previously shown to be sufficient (actually, 1.5 samples/symbol was shown to work in
conjunction with a 5-th order Butterworth antialiasing filter [17]). In this way, the results become almost insensitive to the sampling time error, since the FSE synthesizes, via its transfer characteristic, the necessary phase adjustment [10]. Linear equalization follows, by performing convolution with a bank of four complex valued T/2 spaced FIR filters, arranged in a butterfly structure:

![Equalizer butterfly structure](image)

Figure 2.3: Equalizer butterfly structure

The linear equalizer takes blocks of samples of the incoming signal (equal to the equalizer length) and calculates the minimum-mean-squared-error (MMSE) estimate of the \( k \)th symbol \( \tilde{x}_k \). The optimum solution for the coefficients is obtainable by the Wiener-Hopf equation [17]. However, in practice, \( H \) is time-varying due to PMD, and an adaptive equalizer is necessary. Therefore, the least mean square (LMS) or the recursive least squares (RLS) algorithms can be used to update the coefficients, and track the time varying minimum of the cost function. The LMS coefficient update equation is given by:

\[
W_{k+1} = W_k + \mu \cdot e_k \cdot y_k
\]  

(2.14)

where \( W \) is the \( 2 \times 2 \) equalizer coefficient matrix, \( \mu \) is the algorithm step size, with \( e_k \) and \( y_k \) defined in the form:

\[
e_k = \begin{bmatrix} e_{1,k} \\ e_{2,k} \end{bmatrix} \quad y_k = \begin{bmatrix} y_{1,k} \\ y_{2,k} \end{bmatrix}
\]

(2.15)

Ip and Kahn showed that a linear FSE can compensate for any linear propagation impairment in a dually polarized optical system, given sufficient
oversampling rate, pulse shape and number of equalizer taps are used [2]. If the previous conditions are satisfied, any amount of CD and first order PMD can be compensated with less than 2dB penalty (almost entirely due to CD penalty). The filter length \(N\) is directly proportional to the amount of dispersion and satisfies:

\[
|\beta_2|L_{\text{fiber}}R_s^2(M/K) \approx 0.15N
\]

(2.16)

\[
N = \tau_{\text{DGD}}(M/K)T_s
\]

(2.17)

for compensating CD and PMD, respectively, where \(R_s\) is the data-rate, \(M/K\) is the fractional oversampling rate and \(\tau_{\text{DGD}}\) is the DGD. Typically, the required filter length will be imposed by CD, not by PMD. Therefore, using the value of \(N\) dictated by CD considerations is more than enough to compensate for PMD.

### 2.4 Carrier phase estimation

In the above presented canonical model, the multiplication by the phase noise \(e^{j\phi(t)}\) manifests as a rotation of the received constellation. Additionally, phase noise is a Wiener process with temporal correlation, which means the phase at any symbol period is likely to have a value similar to the phases at adjacent symbols, which allows the usage of signal processing techniques in order to mitigate it. Assuming perfect compensation, the general form of the signal at the output of the equalizer, is:

\[
\tilde{x}_k = x_k e^{j\theta_k} + n_k
\]

(2.18)

where \(x_k\) is the complex valued transmitted symbol at the k-th symbol period, \(\theta_k\) is the carrier phase, and \(n_k\) is AWGN. The goal of the phase estimation process is to find \(\theta_k\), which will allow de-rotation of the signal by multiplying it with \(e^{-j\theta_k}\), followed by a symbol-by-symbol detector to find \(x_k\). Ip and Kahn proposed an algorithm [16], which uses a two stage iteration process for finding the carrier phase. The first stage is a soft decision phase estimator, which computes soft estimates of the carrier phase without taking into account temporal correlation. The soft estimate \(\psi_k\) is given by:

\[
\psi_k = \theta_k + n'_k
\]

(2.19)

where \(n'_k\) is the projection of \(n_k\) onto a vector orthogonal to \(x_k e^{j\theta_k}\). Then this soft estimate \(\psi_k\) is passed through the second stage, which is a linear filter
whose output is the MMSE estimate of $\theta_k$. Figure 2.4 shows a diagram where the composition of the soft estimate is detailed. The soft phase estimator can be either non-decision aided (NDA) or decision-directed (DD). The NDA algorithm is especially well suited to M-ary PSK transmission, since raising the received signal to the M-th power eliminates the phase modulation, due to the M-fold rotational symmetry of an M-PSK constellation, allowing $\theta_k$ to be estimated without any symbol decisions. However, this algorithm is asymptotically optimal for low SNR. On the other hand, the decision directed algorithm replaces the known symbols with the output of a decision device, which is asymptotically optimal for high SNR. Additionally, if the system constellation is non-PSK (non-constant-envelope), the DD algorithm must be used. One disadvantage of this algorithm is that it requires an initial estimate ($\tilde{\theta}_k$) of the phase noise in order to find $\psi_k$. 

![Diagram of the soft phase estimation](image)
Chapter 3

Preliminary developments

According to [16], taking into account the temporal correlation of phase noise, it can be shown that the optimum filter for phase estimation is a Wiener filter, which can be approximated by a FIR filter with sufficient number of taps (e.g. M taps). This filter consists of two exponentially decaying sequences that are symmetric about $n = 0$, causal and anti-causal, with an inherent optimum delay of half the filter length. The causal coefficients are given by:

$$w_n = \frac{\alpha r}{1 - \alpha^2} \alpha^n, \quad n \geq 0$$ (3.1)

where $\alpha = (1 + r/2) - \sqrt{(1 + r/2)^2 - 1}$ and $r = \sigma_p^2/\sigma_n^2$. $\sigma_p^2$ is the variance of the phase noise and $\sigma_n^2$ is the variance of the phase associated with Gaussian noise. The parameter $r$ determines the rate of decay of the filter coefficients, which gives decreasing importance to soft estimates made far away from the current symbol.

Additionally, we are considering the problem of combined equalization and carrier phase recovery and tracking. Therefore, the delay of the Wiener filter needs to be optimized, unless we use a blind stochastic gradient algorithm such as the CMA, which allows the decoupling of these tasks. For DD operation, we have found that the delay of the filter should be set to zero in order to achieve the best performance, since the value of the estimated phase is fed back into the equalizer’s error signal. The benefit of having a small delay and subsequently a less accurate phase estimate, is higher than having an accurate estimate and a larger delay.

Furthermore, it can be shown that when the delay is reduced to zero, the Kalman filter, which is a generalization of the Wiener filter, can be employed
instead, with the advantage that it makes the practical implementation much more feasible. While the later is designed to operate on all of the data directly for each estimate, the former instead, recursively conditions the current estimate on all of the past measurements [9]. The complexity is reduced from $M/2 + 1$ multiplications per symbol, $M$ being the length of the FIR Wiener filter, to only 1 multiplication per symbol.

The Kalman recursion is given by:

$$\tilde{\theta}_{k+1} = \tilde{\theta}_k + G(\psi_k - \tilde{\theta}_k) \tag{3.2}$$

and considering that:

$$\psi_k - \tilde{\theta}_k = \angle(\tilde{x}_k) - \angle(d_{ref}) - \tilde{\theta}_k \tag{3.3}$$

$$= \angle(\tilde{x}_ke^{-j\tilde{\theta}_k} \cdot \text{conj}(d_{ref})) \tag{3.4}$$

Although the gain factor $G$ is supposed to converge during operation, because the inputs are stationary we can pre-compute its steady state value, which can be shown to give

$$G = 1 - \alpha \tag{3.5}$$

and then the Kalman recursion becomes:

$$\tilde{\theta}_{k+1} = \tilde{\theta}_k + G \cdot (\psi_k - \tilde{\theta}_k) \tag{3.6}$$

$$= \tilde{\theta}_k + (1 - \alpha) \cdot \angle(\tilde{x}_ke^{-j\tilde{\theta}_k} \cdot \text{conj}(d_{ref})) \tag{3.7}$$

In Fig. 3.1, the error signal for the equalizer is derived after carrier "de-spin", so that the equalizer output is still a constellation with ringed shape. As the picture shows, the error signal for the equalizer is calculated as follows:

$$e_k = (d_{ref} - \tilde{x}_ke^{-j\tilde{\theta}_k}) \cdot e^{j\tilde{\theta}_k} \tag{3.8}$$

Instead of determining the soft estimate $\psi_k$ [16], here we follow the approach in [18], where the difference $\psi_k - \tilde{\theta}_k$ is calculated directly without the need to perform phase unwrapping. A novel way of calculating the value of $\sigma_n^2$ was introduced. In [16] it is calculated from the SNR. However, as the SNR might be unknown, we do it iteratively, by periodically evaluating the variance of a statistical sufficient window size of the quantity: $\angle(\tilde{x}_ke^{-j\tilde{\theta}_k} \cdot \text{conj}(d_{ref}))$. 

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3.1 Results

In our numerical simulations, we have used a Non-Return to Zero (NRZ) pulse shape \( p(t) \), obtained by passing an ideal rectangular pulse train by a 5th order low pass Bessel filter with a 3-dB bandwidth of 80% of the symbol-rate. For the anti-alias filter \( a(t) \), 3rd order low pass Bessel filters were used.

In Fig. 3.2, the obtained symbol error rate versus input SNR per symbol is shown.

The SNR is defined as:
\[
SNR = \frac{P_x T_s}{N_0} = \frac{E_s}{N_0}
\]

where \( P_x \) is the mean symbol power, \( E_s \) is the symbol energy and \( N_0 \) is the noise power spectral density. A constant of \( 10 \log_{10}(\log_2(M)) \) dB is subtracted to the SNR to reference it per bit, where \( M \) is the constellation size. The MMSE result gives the upper limit on the achievable performance, in the MMSE sense, for a given system pulse response (set of transmission and receiver filters) and signal constellation, obtained by calculating the optimum 13-tap set of coefficients that maximizes the output SNR [17]. The other two results also consider the system pulse response and were obtained by 13-tap adaptive filters (step size \( \mu = 1 \times 10^{-3} \)), with Monte-Carlo simulations and evaluated after convergence. As expected, the LMS curve represented by “×” closely matches the MMSE result, since it considers the same MSE performance surface, using
the steepest descend method to iteratively track the minimum. The result represented by “o” corresponds to the constant modulus algorithm (CMA). Its performance is fairly good for 4-QAM, but very poor for 16-QAM. In this case the CMA cost function is not optimized since the constellation has 3 types of points having different modulus. Fig. 3.3 shows the performance of the system against a sweep in laser linewidth, for both 4 and 16-QAM transmission considering the mentioned system pulse response. The SNR was kept 1dB above the sensitivity at a BER = $1 \times 10^{-3}$, referred to the MMSE curves of Fig. 3.2. A differential bit encoding scheme was employed in all cases, preventing catastrophic bit error propagation when phase noise is high. While the LMS result was obtained by using the proposed method, where the phase is estimated at the output of the equalizer and then included in the error signal, the CMA result corresponds to using separate equalizer convergence and subsequent ideal phase estimation through the FIR Wiener filter. At large linewidths, we can see that either for 4 or 16-QAM modulations, the performance of both CMA and LMS tend to be the same, although the LMS is always better. Additionally, if the linewidth is not very high, the proposed method gives much better performance, particularly for the 16-QAM case.
3.2 Implementation issues

In order not to compromise the performance of the proposed algorithm, it is expected that the carrier phase does not change significantly over the memory length of the equalizer, and its length should be minimized. A practical implementation of this system could use a long filter for CD compensation (e.g. 512-tap), corresponding to ultra long-haul distances, implemented digitally, as the usage of Dispersion Compensating Fiber (DCF) carries some disadvantages. The coefficients of this filter could be either static or slowly updated, and then a subsequent small filter (e.g. 13-tap) with rapid updates could be used, to mitigate the residual uncompensated CD and track time-varying impairments such as PMD. Furthermore, as the time-scales associated with dispersion and phase noise are different, parallelization techniques can be introduced up to a certain extent in the phase noise estimation algorithm.

Concerning a parallel implementation of the algorithm, a look ahead computation can be used [20] to refer the feedback to a result obtained L symbols before, yielding the following Z transform of the zero lag Wiener filter coef-
coefficients:

\[
W(Z) = \frac{(1 - \alpha) \sum_{k=0}^{L-1} \alpha^k Z^{-k}}{1 - \alpha^L z^{-L}}
\] (3.9)

If we calculate the corresponding difference equation, eventually we will obtain the Kalman recursion:

\[
\tilde{\theta}_{k+1} = (1 - \alpha) \sum_{n=0}^{L-1} \alpha^n \psi_{k-n} + \alpha^L \tilde{\theta}_{k-L}
\] (3.10)

Fig. 3.4 shows the BER against linewidth when using different levels of \( L \). Although in [20] the impact of \( L \) was said to be negligible, it is shown here that in fact there is a significant induced degradation for large laser linewidths.

![Figure 3.4: Bit error rate versus linewidth (1dB above the BER = 1 \times 10^{-3} sensitivity point)](image)

**3.3 Conclusion**

The performance of a phase estimation algorithm operating in feedback with a DD LMS equalizer in coherent optical systems was assessed. It was shown
that the corresponding optimum estimator is given by the zero-lag Wiener filter and that it outperforms the CMA + finite-lag Wiener filter approach. Moreover the laser linewidth requirements of the local oscillator can be relaxed which leading to a cost reduction. An hardware efficient implementation based on the Kalman filter was also presented. The issues concerning the employment of a parallel DSP were discussed, where the degradation was shown to be significant for simultaneous large values of the parallelization factor and laser linewidth.
Chapter 4

Research plan

In terms of future work, the main topics will focus on the investigation of the options for coping with nonlinear impairments, as well as further assess the issues of a parallel digital signal processor implementation.

Recently, a novel technique has been suggested for joint compensation of linear and nonlinear impairments in the fiber, using digital backpropagation (BP) [22]. BP involves solving an inverse nonlinear Schrodinger equation (NLSE) through the fiber to estimate the transmitted signal. The main drawbacks of this technique are related to computational complexity issues and difficulty to apply it when significant PMD is present. However, recent work suggests the usage of a reduced complexity algorithm implementation for BP [21]. This has turned to be a very interesting topic of research. Yet, the compensation of nonlinearities above 25GSymbols/s including the effects of PMD might be unfeasible due to computational complexity. This area seems to be quite challenging, and the main international optical research groups have recently started looking at this topic. Specific milestones are outlined below:

1. Study and develop adequate models for fiber nonlinearities, and select which types should be considered in specific application scenarios, e.g. Wavelength Division Multiplexing.

2. Integrate the investigated models within the complete existing coherent system model.

3. Study the recent approach of simplified backpropagation, in order to
compensate for nonlinearities, and develop suitable algorithms. A conference paper is expected to be written as the outcome of this work.

4. Further investigate the combined effects of fiber nonlinearities and PMD.

5. Experimental validation of all the developed models and algorithms. This should be accomplished at the University College London labs, and contacts have been made to make this possible.


The chronogram in Fig.4.1 reflects the estimated necessary time for completing each of the previously outlined tasks.

![Figure 4.1: Chronogram](image-url)
Bibliography


